Using a Timescale Ensemble on Moving Platforms

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***Abstract*— We show through simulation new issues that appear when creating an ensemble of clocks across moving platforms. Much work has been done on the development of timescale algorithms to create ensembles in various circumstances. Most of this work involves stationary clocks. There is some work involving creating an ensemble across satellites. In particular, we address the need for and method of computing relativistic effects. We also show how measurement latency affects the ensemble, as well as the effects of errors in differential positions and velocities.**

Keywords—timescale; clock ensembles; ranging error; relativistic effects on clocks

I. INTRODUCTION

Timescale algorithms have been used for decades in various circumstances. International Atomic Time (TAI) is generated using the algorithm ALGOS to combine clocks from labs throughout the world and then steering the ensemble time in frequency to primary frequency standards [1]. National timing labs have used algorithms to combine co-located, stationary clocks that are measured against each other continuously with minimal measurement noise [2],[3]. The International GNSS Service (IGS) developed a time-scale algorithm for use with reference clocks used in their network [4]. The input to a timescale is the measured differences between the member clocks and one of them, chosen as the pivot clock. The output of a timescale algorithm is the definition of the resultant ensemble time as the offset of each clock from the ensemble.

Realizing this ensemble time as a signal requires steering at least one of the member clocks using these output values [5]. Little work has been done addressing the possibility of combining clocks on platforms moving independently of each other, possibly with high dynamics [5]. This paper addresses some of the issues in using a time-scale algorithm to generate an ensemble across moving platforms. In particular, we show two effects using clock simulation, and one effect using theoretical computation. First, we show the effect of the latency from measurement to steering a member clock to the ensemble time on the resultant steered signal. Second, we show the effect of error in estimating the differential position of moving platforms on the ensemble time. Measuring clock differences across moving platforms requires accounting for the additional delays between measured clocks due to the differential positions and movement of platforms. Finally, we present the effects due to relativity on the clocks moving at high speeds or in varying gravitational potentials.

II. METHODS/RESULTS

We used simulation in Matlab to generate files representing clock differences that have been altered by effects that might be caused by moving platforms. For clock simulation we use the method presented in [7], at least for flicker phase and flicker frequency noise. For white phase noise, we simply use the random number generator of Matlab to generate Gaussian Normal distributions. For a random-walk in phase, which is the same as white noise in frequency [8], we take the white phase noise and compute partial sums. For a random-walk in frequency we compute double partial sums.

A. Effect of latency

At each measurement cycle the output of a timescale algorithm is the offset of each member clock from the ensemble time generated by the algorithm. These data are the digital values of the offsets. To realize the ensemble time as a signal, one must apply one stream of these offsets to a device that steers an output signal of one of the clocks. This steered signal should then be included as an input to the timescale algorithm, but with zero weight, to determine how well the steering is realizing the ensemble time. There are contradictory requirements for an ideal steering algorithm [5]. Steering aggressively to minimize the resultant offset can cause large frequency changes with every update of the steering value. Steering minimally to maximize the frequency stability of the resultant ensemble time can result in a large error offset of the physical signal from the ensemble time. Further, there is always a delay from the instant of the measurement to the time of steering the clocks. By the time a steering offset is applied, the measured clock has wandered somewhat from the value that originated the steering value.

We generate a timescale ensemble based on simulation of a set of input clocks. Because this is simulation, we know the offset of the ensemble clock from an ideal clock. Hence, we can apply the wander of the ensemble time to determine the error in steering a member clock due to an increased latency from the measurement time to the time the steering is applied. In our simulation we use a latency of 5 s and 10 s.

Fig. 1 shows the Modified Allan Deviation (MDEV) [8] of a simulated diverse set of Rubidium (Rb) cell frequency standards, as well as the MDEV of the resultant ensemble time scale from using the AT2 time scale algorithm [3]. The values of the scale are marked in the figure for averaging times of the minimum time, 1 s, and for 16,384 s. We note that the ensemble time is not strictly better in stability than all of input clocks for all averaging times. Previous publications of results using the AT2 algorithm showed the ensemble time better in frequency stability than all input clocks for all averaging times. It appears this may not be strictly the case for a significantly diverse set of clocks.

Fig. 2 shows the effect of a 5 s latency in the delay from measurement to the steering that realizes the ensemble time as a signal. This figure gives the MDEV of the output ensemble time without latency with the effect with latency superimposed. The 5 s latency adds a short-term noise increase from 1.9e-11 to 2.4e-11 at 1 s. The longer-term stability at 16,384 s is unchanged. Fig. 3 similarly shows the MDEV of the effect of a 10 s latency. Here the stability at 1 s increases from 1.9e-11 to 2.7e-11, and again the longer-term stability at 16,384 s is unchanged.

B. *Effect of differential position error*

To investigate the effect of differential position error on platforms we first start with simulating a set of six similar Rb cell clocks, as shown in the MDEV plot of Fig. 4, along with the output ensemble stability. We note here, with the similar stability across the clocks, that the ensemble stability is generally better than that of all clocks at all averaging times. At the very long term, the uncertainty is large for the MDEV values, hence we cannot be certain of results here. We model differential position error as a periodic effect with a period of 15 minutes, and with amplitudes from a uniform random distribution of 0-16 ns. Fig. 5 shows the resulting MDEV of both the input clocks and the output ensemble. The ensemble timescale is able to filter somewhat the short-term instability due to the added measurement noise. But it is not able to recover the best long-term stability. It appears the instability added due to this measurement noise continues to degrade the stability of the ensemble even out to the long-term behavior. Table 1 summarizes the changes in the MDEV stability of both the input clocks and the output ensemble due to adding the measurement noise.

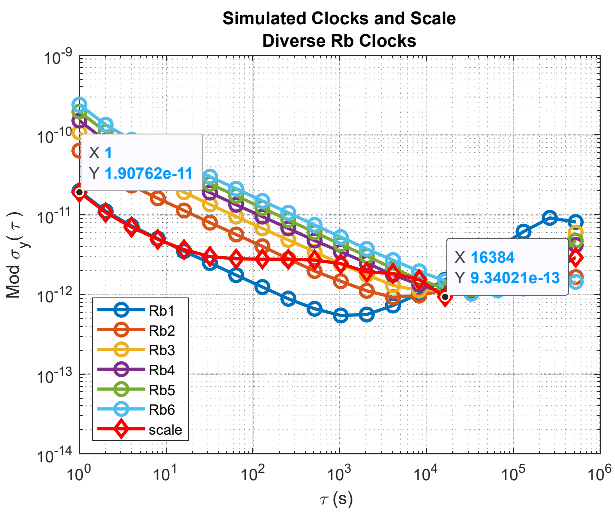

Fig. 1. The Modified Allan Deviation (MDEV) of a simulated set of clocks and the resultant ensemble time. The input clocks simulate the performance of a diverse set of Rubidium cell atomic standards. The output ensemble is generated using the AT2 algorithm.

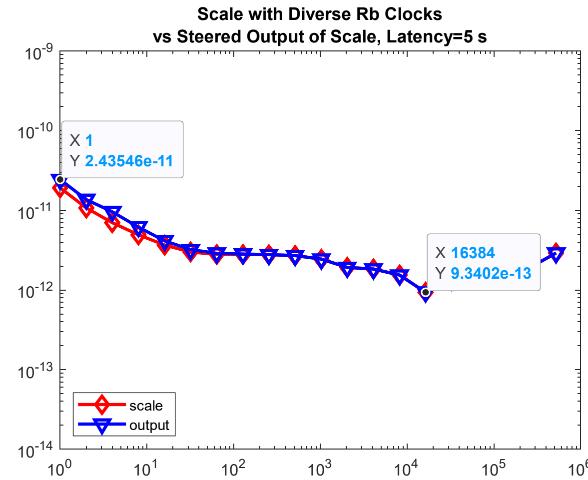

Fig. 2. The effect of a 5 s latency in the delay from measurement to the steering that realizes the ensemble time as a signal. The latency adds a short-term noise increase from 1.9e-11 to 2.4e-11 at 1 stability at 16,384 s is unchanged.

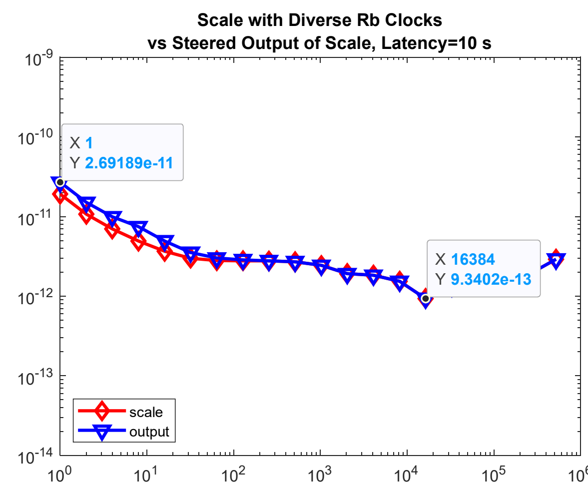

Fig. 3. The effect of a 10 s latency in the delay from measurement to the steering that realizes the ensemble time as a signal. The latency adds a short-term noise increase from 1.9e-11 to 2.7e-11 at 1 s. The stability at 16,384 s is unchanged.

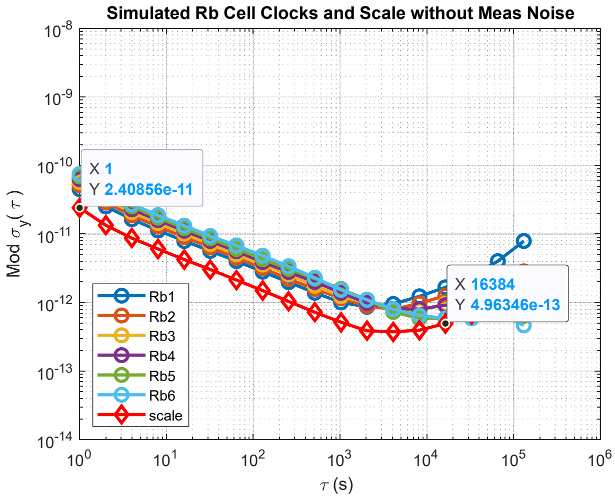

Fig. 4. MDEV of a simulated set of similar Rb cell clocks, plus the output ensemble scale. As before we use the AT2 algorithm.

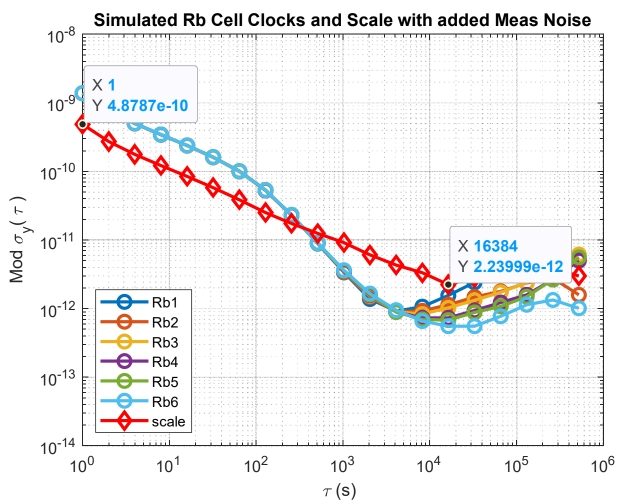

Fig. 5. The output using the same input clocks as Fig. 4, but with added measurement noise modeled as a periodic effect with a period of 15 minutes, and an amplitude as a uniform random variable between 0 and 16 ns. The scale filters the short-term noise somewhat, but the long-term stability of the ensemble scale never reaches that without the measurement noise.

Table 1 Changes in input clock and output ensemble MDEV stability due to adding a periodic measurement noise.

	Change in clock stability		**Change in ensemble stability**	
	From	To	From	To
At 1 s	6.e-11	1.4e-9	2.4e-11	5.e-10
At 4K s	8e-13	1.e-12	3.7e-13	4.3e-12

C. *Relativistic effects*

The special and general theories of relativity state that clocks run slower with higher velocity and faster as they move into a smaller gravitation potential [9]. Identical stationary clocks when moving in different gravitational potentials have different rates and display a different proper time. Ensemble time can be generated when all clocks share the same coordinate time. Therefore, the dynamic clocks that make up the ensemble must be transformed into coordinate time. The platform must measure its own velocity and range with respect to a shared reference point, such as the center of the earth or on the geoid. Using

$$d\tau^2 = \left(1 + \frac{2\varphi}{c^2}\right) dt^2 - \left(1 - \frac{2\varphi}{c^2}\right)\left(\frac{dx^2 + dy^2 + dz^2}{c^2}\right)$$

we can solve for the coordinate time increment, ***dt,*** in terms of $\boldsymbol{d\tau}$, the measured platform proper time. Knowing the gravitational potential and velocity, the platform clock can be transformed from proper time to coordinate time. Once coordinate time is established, we can conduct an initial time transfer to synchronize all clocks in the ensemble. We represent the approach with the following 1-dimensional mathematical description.

$$s_2 = s_0 + \frac{\vec{v} \cdot \vec{D}}{c^2} + \frac{1}{c}\|\vec{D}\|$$

$$t_2 = -s_0 + \frac{1}{c}\|\vec{D}\|$$

$$s_3 = 2\frac{\vec{v} \cdot \vec{D}}{c^2} + \frac{2}{c}\|\vec{D}\|$$

$$t_3 = 2\frac{\vec{v} \cdot \vec{D}}{c^2} + \frac{2}{c}\|\vec{D}\|$$

The unknowns are, ***D***, the distance between two platforms, $\boldsymbol{s_0}$, the clock offset, and ***v*** the relative velocity. $\boldsymbol{s_2}$, $\boldsymbol{t_2}$, and $\boldsymbol{s_3}$ are the times measured on each platform where at least 3 exchanges are required to establish these values. An additional exchange of the measured values is needed for each platform to solve for the unknowns. These are illustrated in Fig. 6.

In matrix form we have

$$\begin{pmatrix} s_2 \\ t_2 \\ s_3 \end{pmatrix} = \begin{bmatrix} 1 & 1/c & 1/c^2 \\ -1 & 1/c & 0 \\ 0 & 1/c & 1/c^2 \end{bmatrix} \begin{pmatrix} s_0 \\ \|\vec{D}\| \\ \vec{v} * \vec{D} \end{pmatrix}$$

where the matrix can be inverted to solve for the unknowns. The $\vec{v} * \vec{D}$ term can be solved for the velocity and reveals the vector nature of the problem. For an actual airborne system this is a 4D vector problem and it requires a communications link between all platforms.

The terms contained in the above equations have picosecond and larger time errors. Terms less than a picosecond have been neglected since these errors are below the fundamental clock noise and the clock corrections occur on timescales faster than these take to accumulate above that noise. A simulation between two airborne systems that had similar 2-hour flight dynamics with noise free clocks, acquired time errors relative to a UTC ground reference of 6.1 ns and 4.1 ns and with differential accumulated time between them of 1.6 ns, as shown in Fig. 7, Fig. 8, and Fig. 9. This algorithm will transform the proper time on each platform to coordinate time. Time transfer and the creation of the ensemble is done using coordinate time.

The same technique can be applied to satellites in a circular orbit, as shown in Fig. 10. The problem is easier since satellite motion is governed by Kepler's Laws. Since each satellite knows the relationship to the leading and following satellite, the communication propagation vector requires very little steering. The systems also know an estimate of the propagation time. Though the $\vec{v} * \vec{D}$ term is much larger than for aircraft, $\vec{v}$ can be estimated by Newton's equations. Orbital and environmental perturbations on each platform will be different and to first order are taken care of with this technique. If we had a cross-link ranging system on GPS, we could implement this technique and have a clock ensemble across them in space.

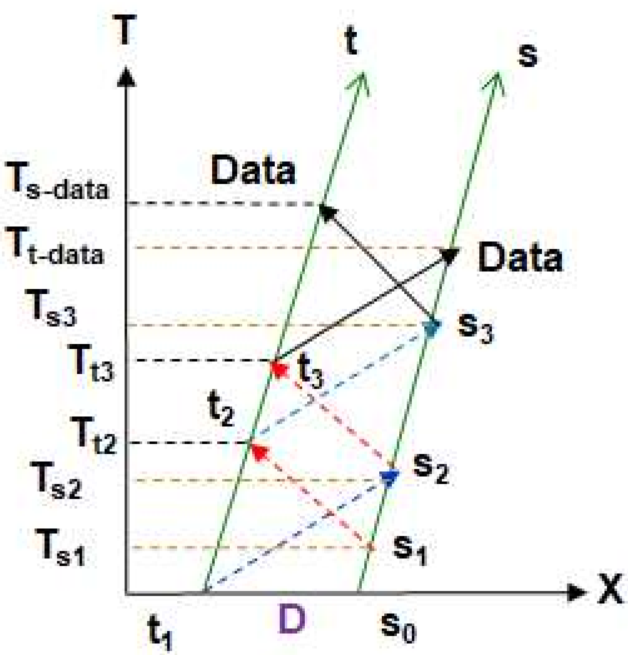

Fig. 6. 1-D time transfer equations with motion correction.

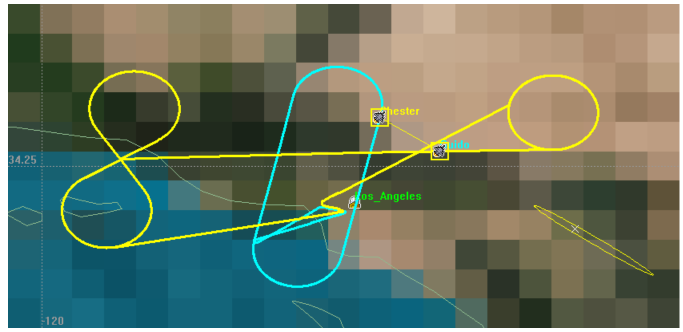

Fig. 7. Simulated flight pattern for relativity test.

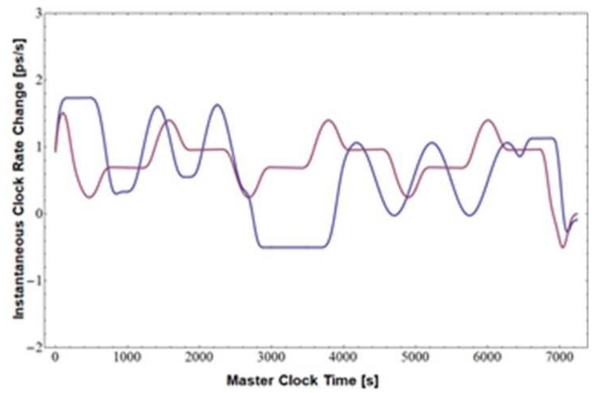

Fig. 8. Clock rate changes due to flights as in Fig. 7.

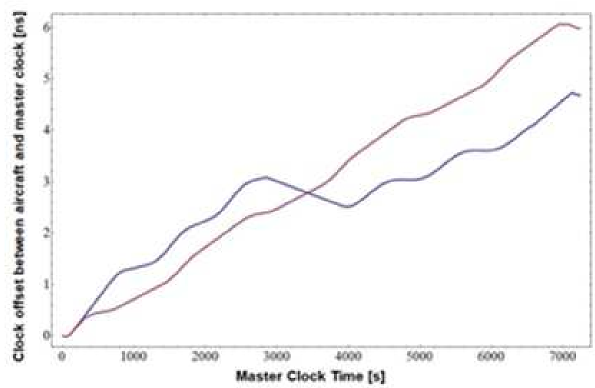

Fig. 9. Clock offsets from master clock due to flight in Fig. 7.

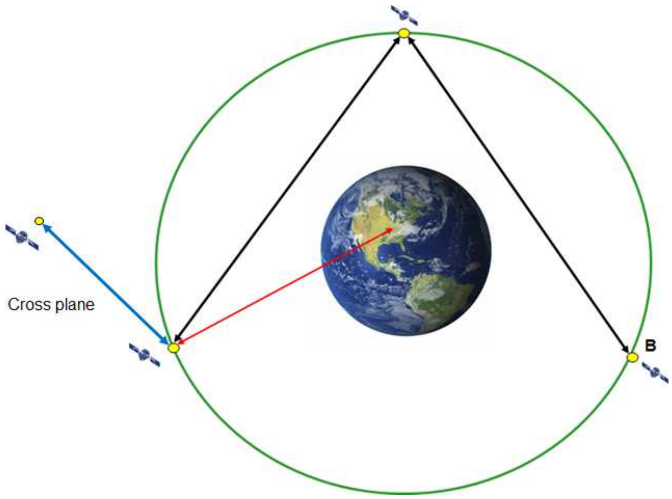

Fig. 10. Satellite constellation synchronization approach.

III. DISCUSSION/INTERPRETATION

A. Effect of latency

One of the first things we find in these results is that the ensemble time is not strictly better in stability than all of input clocks for all averaging times. Many publications that show the stability of the ensemble being better than all the member clocks for all averaging time, use a set of member clocks that have noise types that are more similar than in **Error! Reference source not found.**. It may be that new research is needed to determine optimal algorithms for combining highly diverse clocks.

The main learning from the study of latency is that an increased latency for applying steering to generate the ensemble signal increases the short-term instability. The 5 s latency increases short-term instability from 1.9e-11 to 2.4e-11 at 1 s. The 10 s latency increases it from 1.9e-11 to 2.7e-11. Now a signal traveling at the speed of light moves 1.5e6 km in 5 s. This is large even for a geostationary orbit, which can have a radius of just over 40,000 km. The scale was chosen to be large enough to be visible on an MDEV plot. For systems where milliseconds of stability or better matter, this effect must be considered in the design.

B. Effect of differential position error

Our study of the effect of differential position error on a timescale ensemble shows that persistent ranging error can add significant noise to the ensemble stability. Of course, different systems have different requirements. The take-away here is that differential position accuracy must be matched to system requirements.

C. Relativistic effects

Simulations of relativistic effects using a good rubidium standard in an airborne system showed that after approximately 100 seconds of flight, the errors from relativity were dominant. The relativistic errors accumulated faster than the deviation from clock noise as measured in the laboratory. Relativistic errors are deterministic and as navigation, ranging, and communication systems improve we envision that these errors can be corrected using look-up tables in software.

IV. CONCLUSIONS

Precision clocks are being used on many cyber-physical systems. It may be useful to link some of these clocks and combine them into ensembles using timescale algorithms. In this way, many systems can become more resilient to faults, or to intentional or unintentional interference since a clock in such an ensemble is continuously estimated to maintain its offset from the ensemble time. For example, land, air, and water autonomous vehicles in a domain, such as a city, could be linked by timescale algorithms to create ensembles, if we can develop near-optimal algorithms for such purposes. This paper is an initial step in that direction.

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